Matrices

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1. In the matrix $A = \begin{bmatrix} 2 & amp;5 & amp;19 & amp;-7 \\ 35 & amp;-2 & amp;\frac{5}{2} & amp;12 \\ \sqrt{3} & amp;1 & amp;-5 & amp;17 \end{bmatrix}$,write :
   (i) The order of the matrix
   (ii) The number of elements,
   (iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

   ### Solutions :

   (i) In the given matrix, the number of rows is 3 and the number of columns is 4. Therefore, the order of the matrix is $3 * 4$
   . (ii) Since the order of the matrix is $3 * 4$, there are $3 * 4 = 12$ elements in it.
   (iii)$a_{13} = 19, a_{21} = 35, a_{33} = -5, a_{24} = 12, a_{23} = \frac{5}{2}$

2. If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

   ### Solutions :

   We know that if a matrix is of the order $m * n$, it has mn elements.
   Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.
   The ordered pairs are: $(1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), and (6, 4)$
   Hence, the possible orders of a matrix having 24 elements are:
   $1 * 24, 24 * 1, 2 * 12, 12 * 2,$
   $3 * 8, 8 * 3, 4 * 6$ and $6 * 4$
   $(1, 13)$ and $(13, 1)$ are the ordered pairs of natural numbers whose product is 13.
   Hence the possible orders of matrix having 13 elements are (1* 13) and (13 * 1).

3. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

   ### Solutions :

   We know that if a matrix is of the order $m * n$, it has mn elements.
   Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose products is 18.
   The ordered pairs are:$(1, 18), (18, 1), (2, 9), (9, 2), (3, 6), and (6, 3)$
   Hence, the possible orders of a matrix having 18 elements are :
   $1 * 18, 18 * 1, 2 * 9, 9 * 2, 3 * 6$ and $6 * 3$
   $(1, 5)$ and $(5, 1)$ are the ordered pairs of natural numbers whose product is 5.
   Hence, the possible orders of a matrix having 5 elements are $1 * 5$ and $5 * 1$.

4. Construct a $3x4$ matrix, whose elements are given by
   $(i) a_{ij} = \frac{1}{2}|-3i + j|$
   $(ii) a_{ij} = 2i - j$

   ### Solutions :

   In general, a $3x4$ matrix is given by $A = \begin{bmatrix} a_{11} & amp;a_{12} & amp;a_{13} & amp;a_{14} \\ a_{21} & amp;a_{22} & amp;a_{23} & amp;a_{24} \\ a_{31} & amp;a_{32} & amp;a_{33} & amp;a_{34} \end{bmatrix}$
   (i) Given : $a_{ij} = \frac{1}{2}|-3i + j|, i = 1, 2, 3$ and $j = 1, 2, 3, 4$
   Thus , we have
   $a_{11} = \frac{1}{2}|-3*1+1| = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = \frac{2}{2} = 1$
   $a_{21} = \frac{1}{2}|-3*2+1| = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{5}{2}$
   $a_{31} = \frac{1}{2}|-3*3+1| = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = 4$
   $a_{12} = \frac{1}{2}|-3*1+2| = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}$
   $a_{22} = \frac{1}{2}|-3*2+2| = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$
   $a_{32} = \frac{1}{2}|-3*3+2| = \frac{1}{2}|-9+2| = \frac{1}{2}|-7| = \frac{7}{2}$
   $a_{13} = \frac{1}{2}|-3*1+3| = \frac{1}{2}|-3+3| = 0$
   $a_{23} = \frac{1}{2}|-3*2+3| = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{3}{2}$
   $a_{33} = \frac{1}{2}|-3*3+3| = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$
   $a_{14} = \frac{1}{2}|-3*1+4| = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}$
   $a_{24} = \frac{1}{2}|-3*2+4| = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$

$a_{34} = \frac{1}{2}|-3*3+4| = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{5}{2}$

Therefore , the required matrix is $A = \begin{bmatrix} 1 & amp; \frac{1}{2} & amp; 0 & amp; \frac{1}{2} \\ \frac{5}{2} & amp; 2 & amp; \frac{3}{2} & amp; 1 \\ 4 & amp; \frac{7}{2} & amp; 3 & amp; \frac{5}{2} \end{bmatrix}$

(ii) Given :

$a_{ij} = 2i - j, i = 1, 2, 3$ and j = 1, 2,3, 44

Thus, we have

- $a_{11} = 2*1-1 = 2-1 = 1$
- $a_{21} = 2*2-1 = 4-1 = 3$
- $a_{31} = 2*3-1 = 6-1 = 5$
- $a_{12} = 2*1-2 = 2-2 = 0$
- $a_{22} = 2*2-2 = 4-2 = 2$
- $a_{32} = 2*3-2 = 6-2 = 4$
- $a_{13} = 2*1-3 = 2-3 = -1$
- $a_{23} = 2*2-3 = 4-3 = 1$
- $a_{33} = 2*3-3 = 6-3 = 3$
- $a_{14} = 2*1-4 = 2-4 = -2$
- $a_{24} = 2*2-4 = 4-4 = 0$
- $a_{34} = 2*3-4 = 6-4 = 2$

Therefore ,the required matrix is $A = \begin{bmatrix} 1 & amp; 0 & amp; -1 & amp; -2 \\ 3 & amp; 2 & amp; 1 & amp; 0 \\ 5 & amp; 4 & amp; 3 & amp; 2 \end{bmatrix}$

$a_{13} = \frac{1}{2}|-3*1+3| = \frac{1}{2}|-3+3| = 0$

$a_{23} = \frac{1}{2}|-3*2+3| = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{3}{2}$

$a_{33} = \frac{1}{2}|-3*3+3| = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$

$a_{14} = \frac{1}{2}|-3*1+4| = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}$

$a_{24} = \frac{1}{2}|-3*2+4| = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$

$a_{34} = \frac{1}{2}|-3*3+4| = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{5}{2}$

Therefore , the required matrix is $A = \begin{bmatrix} 1 & amp; \frac{1}{2} & amp; 0 & amp; \frac{1}{2} \\ \frac{5}{2} & amp; 2 & amp; \frac{3}{2} & amp; 1 \\ 4 & amp; \frac{7}{2} & amp; 3 & amp; \frac{5}{2} \end{bmatrix}$

5. Find the value of x, y and z from the following equation:

$(i) \begin{bmatrix} 4 & amp; 3 \\ x & amp; 5 \end{bmatrix} = \begin{bmatrix} y & amp; z \\ 1 & amp; 5 \end{bmatrix}$ $(ii) \begin{bmatrix} x+y & amp; 2 \\ 5+z & amp; xy \end{bmatrix} = \begin{bmatrix} 6 & amp; 2 \\ 5 & amp; 8 \end{bmatrix}$ $(iii) \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Solutions :

(i) Given : $\begin{bmatrix} 4 & amp; 3 \\ x & amp; 5 \end{bmatrix} = \begin{bmatrix} y & amp; z \\ 1 & amp; 5 \end{bmatrix}$

As the given matrices are equal, their corresponding elements are also equal.Comparing the corresponding elements, we get:

$x = 1, y = 4$and $z = 3$

$(iii) \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$x + y + z = 9 ...(1)$

$x + z = 5 ...(2)$

$y + z = 7 ...(3)$

From (1) and (2), we have:

$y + 5 = 9 \implies y = 4$

From (3), we have:

$4 + z = 7 \implies z = 3$

$\therefore x + z = 5 \implies x = 2$

$\therefore x = 2, y = 4,$ and $z = 3$

$(ii) \begin{bmatrix} x+y & amp; 2 \\ 5+z & amp; xy \end{bmatrix} = \begin{bmatrix} 6 & amp; 2 \\ 5 & amp; 8 \end{bmatrix}$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$x+y=6, xy=8, 5+z=5$ Now,
$5+z=5 \implies z=0$
Using $(x-y)^2=(x+y)^2-4xy$, we get
$\implies (x-y)^2=36-32=4$
$\implies x-y=\pm 2$
When x - y = 2 and x + y =6 ,we get x = 4 and y = 2
When x - y =- 2 and x + y = 6 we get x = 2 and y = 4
∴ x =4, y = 2, and z = 0 or x = 2 , y = 4 , and z = 0

6. Find the value of a, b , c , and d from the equation:
$$\begin{bmatrix} a-b & amp; 2a+c \\ 2a-b & amp; 3c+d \end{bmatrix} = \begin{bmatrix} -1 & amp; 5 \\ 0 & amp; 13 \end{bmatrix}$$

Solutions :

$$\begin{bmatrix} a-b & amp; 2a+c \\ 2a-b & amp; 3c+d \end{bmatrix} = \begin{bmatrix} -1 & amp; 5 \\ 0 & amp; 13 \end{bmatrix}$$
As the two matrices are equal, their corresponding elements are also equal. Comparing the corresponding elements, we get:
$a-b=-1$........(1)
$2a-b=0$........(2)
$2a+c=5$.......(3)
$3c+d=13$
From (2), we have:
$b=2a$
Then, from (1), we have:
$a-2a=-1$
$\implies a=1$
$\implies b=2$
Now, from (3), we have:
$2*1=c-5$
$\implies c=3$
From (4) we have:
$3*3+d=13$
$\implies 9+d=13 \implies d=4$
$\therefore a=1, b=2, c=3$, and $d=4$

7. $A=\left[a_{ij}\right]_{m*n}$ is a square matrix, if

Solutions :

The correct answer is C.
It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.
Therefore, $A=\left[a_{ij}\right]_{m*n}$ is a square matrix, if $m=n$.

8. Which of the given values of x and y make the following pair of matrices equal
$$\begin{bmatrix} 3x+7 & amp; 5 \\ y+1 & amp; 2-3x \end{bmatrix} = \begin{bmatrix} 0 & amp; y-2 \\ 8 & amp; 4 \end{bmatrix}$$

Solutions :

The Correct answer is B.
It is given that $\begin{bmatrix} 3x+7 & amp; 5 \\ y+1 & amp; 2-3x \end{bmatrix} = \begin{bmatrix} 0 & amp; y-2 \\ 8 & amp; 4 \end{bmatrix}$ Equating the corresponding elements, we get:
$3x+7=0 \implies x=\frac{-7}{3}$
$5=y-2 \implies y=7$
$y+1=8 \implies y=7$
$2-3x=4 \implies x=-\frac{2}{3}$
We find that on comparing the corresponding elements of the two matrices, we get two different values of x, which is not possible.
Hence, it is not possible to find the values of x and y for which the given matrices are equal.

9. The number of all possible matrices of order $3*3$ with each entry 0 or 1 is:

Solutions :

The correct answer is D. The given matrix of the order $3*3$ has 9 elements and each of these elements can be either 0 or 1.
Now, each of the 9 elements can be filled in two possible ways. Therefore, by the multiplication principle, the required number of possible matrices is $2^9=512$

10. Let $A=\begin{bmatrix} 2 & amp; 4 \\ 3 & amp; 2 \end{bmatrix}, B=\begin{bmatrix} 1 & amp; 3 \\ -2 & amp; 5 \end{bmatrix}, C=\begin{bmatrix} -2 & amp; 5 \\ 3 & amp; 4 \end{bmatrix}$
Find each of the following
$(i) A+B$ $\quad (ii) A-B$
$(iii) 3A-C$ $\quad (iv) AB$
$(v) BA$

Solutions :

$(i) A+B=\begin{bmatrix} 2 & amp; 4 \\ 3 & amp; 2 \end{bmatrix}+\begin{bmatrix} 1 & amp; 3 \\ -2 & amp; 5 \end{bmatrix}=\begin{bmatrix} 2+1 & amp; 4+3 \\ 3-2 & amp; 2+5 \end{bmatrix}=\begin{bmatrix} 3 & amp; 7 \\ 1 & amp; 7 \end{bmatrix}$

$(i) A-B=\begin{bmatrix} 2 & amp; 4 \\ 3 & amp; 2 \end{bmatrix}-\begin{bmatrix} 1 & amp; 3 \\ -2 & amp; 5 \end{bmatrix}=\begin{bmatrix} 2-1 & amp; 4-3 \\ 3-(-2) & amp; 2-5 \end{bmatrix}=\begin{bmatrix} 1 & amp; 1 \\ 5 & amp; -3 \end{bmatrix}$

$(iii) 3A - C = 3\begin{bmatrix}2 & amp;4\\3 & amp;2\end{bmatrix} - \begin{bmatrix}-2 & amp;5\\3 & amp;4\end{bmatrix} = \begin{bmatrix}3*2 & amp;3*4\\3*3 & amp;3*2\end{bmatrix} - \begin{bmatrix}-2 & amp;5\\3 & amp;4\end{bmatrix}$

$= \begin{bmatrix}6 & amp;12\\9 & amp;6\end{bmatrix} - \begin{bmatrix}-2 & amp;5\\3 & amp;4\end{bmatrix}$

$\begin{bmatrix}6+2 & amp;12-5\\9-3 & amp;6-4\end{bmatrix}$

$\begin{bmatrix}8 & amp;7\\6 & amp;2\end{bmatrix}$

(iv) Matrix A has 2 columns. This number is equal to the number of rows in matrix B. Therefore, AB is defined as:

$AB = \begin{bmatrix}2 & amp;4\\3 & amp;2\end{bmatrix}\begin{bmatrix}1 & amp;3\\-2 & amp;5\end{bmatrix}$

$\begin{bmatrix}2(1)+4(-2) & amp;2(3)+4(5)\\3(1)+2(-2) & amp;3(3)+2(5)\end{bmatrix}$

$\begin{bmatrix}2-8 & amp;6+20\\3-4 & amp;9+10\end{bmatrix}$

$\begin{bmatrix}-6 & amp;26\\-1 & amp;19\end{bmatrix}$

(v) matrix B has 2 columns. This number is equal to the number of rows in matrix A. Therefore, BA is defined as:

$BA = \begin{bmatrix}1 & amp;3\\-2 & amp;5\end{bmatrix}\begin{bmatrix}2 & amp;4\\3 & amp;2\end{bmatrix}$

$= \begin{bmatrix}1(2)+3(3) & amp;1(4)+3(2)\\-2(2)+5(3) & amp;-2(4)+5(2)\end{bmatrix}$

$= \begin{bmatrix}2+9 & amp;4+6\\-4+15 & amp;-8+10\end{bmatrix} = \begin{bmatrix}11 & amp;10\\11 & amp;2\end{bmatrix}$

11. Compute the following:

$(i) \begin{bmatrix}a & amp;b\\-b & amp;a\end{bmatrix} + \begin{bmatrix}a & amp;b\\b & amp;a\end{bmatrix}$

$(ii) \begin{bmatrix}a^2+b^2 & amp;b^2+c^2\\a^2+c^2 & amp;a^2+b^2\end{bmatrix} + \begin{bmatrix}2ab & amp;2bc\\-2ac & amp;-2ab\end{bmatrix}$

$(iii) \begin{bmatrix}-1 & amp;4 & amp;-6\\8 & amp;5 & amp;16\\2 & amp;8 & amp;5\end{bmatrix} + \begin{bmatrix}12 & amp;7 & amp;6\\8 & amp;0 & amp;5\\3 & amp;2 & amp;4\end{bmatrix}$

$(iv) \begin{bmatrix}\cos^2 x & amp;\sin^2 x\\\sin^2 x & amp;\cos^2 x\end{bmatrix} + \begin{bmatrix}\sin^2 x & amp;\cos^2 x\\\cos^2 x & amp;\sin^2 x\end{bmatrix}$

Solutions :

$(i) \begin{bmatrix}a & amp;b\\-b & amp;a\end{bmatrix} + \begin{bmatrix}a & amp;b\\b & amp;a\end{bmatrix} = \begin{bmatrix}a+a & amp;b+b\\-b+b & amp;a+a\end{bmatrix} = \begin{bmatrix}2a & amp;2b\\0 & amp;2a\end{bmatrix}$

$(ii) \begin{bmatrix}a^2+b^2 & amp;b^2+c^2\\a^2+c^2 & amp;a^2+b^2\end{bmatrix} + \begin{bmatrix}2ab & amp;2bc\\-2ac & amp;-2ab\end{bmatrix}$

$= \begin{bmatrix}a^2+b^2+2ab & amp;b^2+c^2+2bc\\a^2+c^2-2ac & amp;a^2+b^2-2ab\end{bmatrix}$

$= \begin{bmatrix}(a+b)^2 & amp;(b+c)^2\\(a-c)^2 & amp;(a-b)^2\end{bmatrix}$

$(iii) \begin{bmatrix}-1 & amp;4 & amp;-6\\8 & amp;5 & amp;16\\2 & amp;8 & amp;5\end{bmatrix} + \begin{bmatrix}12 & amp;7 & amp;6\\8 & amp;0 & amp;5\\3 & amp;2 & amp;4\end{bmatrix}$

$= \begin{bmatrix}-1+12 & amp;4+7 & amp;-6+6\\8+8 & amp;5+0 & amp;16+5\\2+3 & amp;8+2 & amp;5+4\end{bmatrix}$

$= \begin{bmatrix}11 & amp;11 & amp;0\\16 & amp;5 & amp;21\\5 & amp;10 & amp;9\end{bmatrix}$

$(iv) \begin{bmatrix}\cos^2 x & amp;\sin^2 x\\\sin^2 x & amp;\cos^2 x\end{bmatrix} + \begin{bmatrix}\sin^2 x & amp;\cos^2 x\\\cos^2 x & amp;\sin^2 x\end{bmatrix}$

$= \begin{bmatrix}\cos^2 x+\sin^2 x & amp;\sin^2 x+\cos^2 x\\\sin^2 x+\cos^2 x & amp;\cos^2 x+\sin^2 x\end{bmatrix}$

$= \begin{bmatrix}1 & amp;1\\1 & amp;1\end{bmatrix} \quad (\therefore \cos^2 x = 1)$

$(iii) \begin{bmatrix}-1 & amp;4 & amp;-6\\8 & amp;5 & amp;16\\2 & amp;8 & amp;5\end{bmatrix} + \begin{bmatrix}12 & amp;7 & amp;6\\8 & amp;0 & amp;5\\3 & amp;2 & amp;4\end{bmatrix}$

$$= \begin{bmatrix} -1+12 & amp;4+7 & amp;-6+6 \\ 8+8 & amp;5+0 & amp;16+5 \\ 2+3 & amp;8+2 & amp;5+4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & amp;11 & amp;0 \\ 16 & amp;5 & amp;21 \\ 5 & amp;10 & amp;9 \end{bmatrix}$$

$$(ii) \begin{bmatrix} a^2+b^2 & amp;b^2+c^2 \\ a^2+c^2 & amp;a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & amp;2bc \\ -2ac & amp;-2ab \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2+2ab & amp;b^2+c^2+2bc \\ a^2+c^2-2ac & amp;a^2+b^2-2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 & amp;(b+c)^2 \\ (a-c)^2 & amp;(a-b)^2 \end{bmatrix}$$

$$(iv) \begin{bmatrix} \cos^2 x & amp;\sin^2 x \\ \sin^2 x & amp;\cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & amp;\cos^2 x \\ \cos^2 x & amp;\sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x+\sin^2 x & amp;\sin^2 x+\cos^2 x \\ \sin^2 x+\cos^2 x & amp;\cos^2 x+\sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & amp;1 \\ 1 & amp;1 \end{bmatrix} \quad (\therefore \cos^2 x = 1)$$

12. Compute the indicated products

$$(i) \begin{bmatrix} a & amp;b \\ -b & amp;a \end{bmatrix} \begin{bmatrix} a & amp;-b \\ b & amp;a \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & amp;3 & amp;4 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 1 & amp;-2 \\ 2 & amp;3 \end{bmatrix} \begin{bmatrix} 1 & amp;2 & amp;3 \\ 2 & amp;3 & amp;1 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 2 & amp;3 & amp;4 \\ 3 & amp;4 & amp;5 \\ 4 & amp;5 & amp;6 \end{bmatrix} \begin{bmatrix} 1 & amp;-3 & amp;5 \\ 0 & amp;2 & amp;4 \\ 3 & amp;0 & amp;5 \end{bmatrix}$$

$$(v) \begin{bmatrix} 2 & amp;1 \\ 3 & amp;2 \\ -1 & amp;1 \end{bmatrix} \begin{bmatrix} 1 & amp;0 & amp;1 \\ -1 & amp;2 & amp;1 \end{bmatrix}$$

$$(vi) \begin{bmatrix} 3 & amp;-1 & amp;3 \\ -1 & amp;0 & amp;2 \end{bmatrix} \begin{bmatrix} 2 & amp;-3 \\ 1 & amp;0 \\ 3 & amp;1 \end{bmatrix}$$

Solutions :

$$(i) \begin{bmatrix} a & amp;b \\ -b & amp;a \end{bmatrix} \begin{bmatrix} a & amp;-b \\ b & amp;a \end{bmatrix}$$

$$= \begin{bmatrix} a(a)+b(b) & amp;a(-b)+b(a) \\ -b(a)+a(b) & amp;-b(-b)+a(a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & amp;-ab+ab \\ -ab+ab & amp;b^2+a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & amp;0 \\ 0 & amp;a^2+b^2 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & amp;3 & amp;4 \end{bmatrix}$$

$$= \begin{bmatrix} 1(2) & amp;1(3) & amp;1(4) \\ 2(2) & amp;2(3) & amp;2(4) \\ 3(2) & amp;3(3) & amp;3(4) \end{bmatrix} = \begin{bmatrix} 2 & amp;3 & amp;4 \\ 4 & amp;6 & amp;8 \\ 6 & amp;9 & amp;12 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 1 & amp;-2 \\ 2 & amp;3 \end{bmatrix} \begin{bmatrix} 1 & amp;2 & amp;3 \\ 2 & amp;3 & amp;1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)-2(2) & amp;1(2)-2(3) & amp;1(3)-2(1) \\ 2(1)+3(2) & amp;2(2)+3(3) & amp;2(3)+3(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & amp;2-6 & amp;3-2 \\ 2+6 & amp;4+9 & amp;6+3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & amp;-4 & amp;1 \\ 8 & amp;13 & amp;9 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 2 & amp;3 & amp;4 \\ 3 & amp;4 & amp;5 \\ 4 & amp;5 & amp;6 \end{bmatrix} \begin{bmatrix} 1 & amp;-3 & amp;5 \\ 0 & amp;2 & amp;4 \\ 3 & amp;0 & amp;5 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1)+3(0)+4(3) \quad amp; 2(-3)+3(2)+4(0) \quad amp; 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) \quad amp; 3(-3)+4(2)+5(0) \quad amp; 3(5)+4(4)+5(5) \\ 4(1)+5(0)+6(3) \quad amp; 4(-3)+5(2)+6(0) \quad amp; 4(5)+5(4)+6(5) \end{bmatrix}$$
$$= \begin{bmatrix} 2+0+12 \quad amp; -6+6+0 \quad amp; 10+12+20 \\ 3+0+15 \quad amp; -9+8+0 \quad amp; 15+16+25 \\ 4+0+18 \quad amp; -12+10+0 \quad amp; 20+20+30 \end{bmatrix}$$
$$= \begin{bmatrix} 14 \quad amp; 0 \quad amp; 42 \\ 18 \quad amp; -1 \quad amp; 56 \\ 22 \quad amp; -2 \quad amp; 70 \end{bmatrix}$$

$$(v) \begin{bmatrix} 2 \quad amp; 1 \\ 3 \quad amp; 2 \\ -1 \quad amp; 1 \end{bmatrix} \begin{bmatrix} 1 \quad amp; 0 \quad amp; 1 \\ -1 \quad amp; 2 \quad amp; 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2(1)+1(-1) \quad amp; 2(0)+1(2) \quad amp; 2(1)+1(1) \\ 3(1)+2(-1) \quad amp; 3(0)+2(2) \quad amp; 3(1)+2(1) \\ -1(1)+1(-1) \quad amp; -1(0)+1(2) \quad amp; -(1)+1(1) \end{bmatrix}$$
$$= \begin{bmatrix} 2-1 \quad amp; 0+2 \quad amp; 2+1 \\ 3-2 \quad amp; 0+4 \quad amp; 3+2 \\ -1-1 \quad amp; 0+2 \quad amp; -1+1 \end{bmatrix} = \begin{bmatrix} 1 \quad amp; 2 \quad amp; 3 \\ 1 \quad amp; 4 \quad amp; 5 \\ -2 \quad amp; 2 \quad amp; 0 \end{bmatrix}$$

$$(vi) \begin{bmatrix} 3 \quad amp; -1 \quad amp; 3 \\ -1 \quad amp; 0 \quad amp; 2 \end{bmatrix} \begin{bmatrix} 2 \quad amp; -3 \\ 1 \quad amp; 0 \\ 3 \quad amp; 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3(2)-1(1)+3(3) \quad amp; 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) \quad amp; -1(-3)+0(0)+2(1) \end{bmatrix}$$
$$= \begin{bmatrix} 6-1+9 \quad amp; -9-0+3 \\ -2+0+6 \quad amp; 3+0+2 \end{bmatrix} = \begin{bmatrix} 14 \quad amp; -6 \\ 4 \quad amp; 5 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 1 \quad amp; -2 \\ 2 \quad amp; 3 \end{bmatrix} \begin{bmatrix} 1 \quad amp; 2 \quad amp; 3 \\ 2 \quad amp; 3 \quad amp; 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1(1)-2(2) \quad amp; 1(2)-2(3) \quad amp; 1(3)-2(1) \\ 2(1)+3(2) \quad amp; 2(2)+3(3) \quad amp; 2(3)+3(1) \end{bmatrix}$$
$$= \begin{bmatrix} 1-4 \quad amp; 2-6 \quad amp; 3-2 \\ 2+6 \quad amp; 4+9 \quad amp; 6+3 \end{bmatrix}$$
$$= \begin{bmatrix} -3 \quad amp; -4 \quad amp; 1 \\ 8 \quad amp; 13 \quad amp; 9 \end{bmatrix}$$

$$(v) \begin{bmatrix} 2 \quad amp; 1 \\ 3 \quad amp; 2 \\ -1 \quad amp; 1 \end{bmatrix} \begin{bmatrix} 1 \quad amp; 0 \quad amp; 1 \\ -1 \quad amp; 2 \quad amp; 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2(1)+1(-1) \quad amp; 2(0)+1(2) \quad amp; 2(1)+1(1) \\ 3(1)+2(-1) \quad amp; 3(0)+2(2) \quad amp; 3(1)+2(1) \\ -1(1)+1(-1) \quad amp; -1(0)+1(2) \quad amp; -(1)+1(1) \end{bmatrix}$$
$$= \begin{bmatrix} 2-1 \quad amp; 0+2 \quad amp; 2+1 \\ 3-2 \quad amp; 0+4 \quad amp; 3+2 \\ -1-1 \quad amp; 0+2 \quad amp; -1+1 \end{bmatrix} = \begin{bmatrix} 1 \quad amp; 2 \quad amp; 3 \\ 1 \quad amp; 4 \quad amp; 5 \\ -2 \quad amp; 2 \quad amp; 0 \end{bmatrix}$$

$$(vi) \begin{bmatrix} 3 \quad amp; -1 \quad amp; 3 \\ -1 \quad amp; 0 \quad amp; 2 \end{bmatrix} \begin{bmatrix} 2 \quad amp; -3 \\ 1 \quad amp; 0 \\ 3 \quad amp; 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3(2)-1(1)+3(3) \quad amp; 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) \quad amp; -1(-3)+0(0)+2(1) \end{bmatrix}$$
$$= \begin{bmatrix} 6-1+9 \quad amp; -9-0+3 \\ -2+0+6 \quad amp; 3+0+2 \end{bmatrix} = \begin{bmatrix} 14 \quad amp; -6 \\ 4 \quad amp; 5 \end{bmatrix}$$

13. If $A = \begin{bmatrix} 1 \quad amp; 2 \quad amp; -3 \\ 5 \quad amp; 0 \quad amp; 2 \\ 1 \quad amp; -1 \quad amp; 1 \end{bmatrix}$,$B = \begin{bmatrix} 3 \quad amp; -1 \quad amp; 2 \\ 4 \quad amp; 2 \quad amp; 5 \\ 2 \quad amp; 0 \quad amp; 3 \end{bmatrix}$,and $C = \begin{bmatrix} 4 \quad amp; 1 \quad amp; 2 \\ 0 \quad amp; 3 \quad amp; 2 \\ 1 \quad amp; -2 \quad amp; 3 \end{bmatrix}$,then Compute $(A+B)$ and $(B-C)$. Also , verify that $A+(B-C) = (A+B)-C$

Solutions :

$$A+B = \begin{bmatrix} 1 \quad amp; 2 \quad amp; -3 \\ 5 \quad amp; 0 \quad amp; 2 \\ 1 \quad amp; -1 \quad amp; 1 \end{bmatrix} + \begin{bmatrix} 3 \quad amp; -1 \quad amp; 2 \\ 4 \quad amp; 2 \quad amp; 5 \\ 2 \quad amp; 0 \quad amp; 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1+3 \quad amp; 2-1 \quad amp; -3+2 \\ 5+4 \quad amp; 0+2 \quad amp; 2+5 \\ 1+2 \quad amp; -1+0 \quad amp; 1+3 \end{bmatrix} = \begin{bmatrix} 4 \quad amp; 1 \quad amp; -1 \\ 9 \quad amp; 2 \quad amp; 7 \\ 3 \quad amp; -1 \quad amp; 4 \end{bmatrix}$$
$$B-C = \begin{bmatrix} 3 \quad amp; -1 \quad amp; 2 \\ 4 \quad amp; 2 \quad amp; 5 \\ 2 \quad amp; 0 \quad amp; 3 \end{bmatrix} - \begin{bmatrix} 4 \quad amp; 1 \quad amp; 2 \\ 0 \quad amp; 3 \quad amp; 2 \\ 1 \quad amp; -2 \quad amp; 3 \end{bmatrix}$$

$$A+(B-C)=\begin{bmatrix}1 & amp;2 & amp;-3\\5 & amp;0 & amp;2\\1 & amp;-1 & amp;1\end{bmatrix}+\begin{bmatrix}-1 & amp;-2 & amp;0\\4 & amp;-1 & amp;3\\1 & amp;2 & amp;0\end{bmatrix}$$

$$\begin{bmatrix}1+(-1) & amp;2+(-2) & amp;-3+0\\5+4 & amp;0+(-1) & amp;2+3\\1+1 & amp;-1+2 & amp;1+0\end{bmatrix}=\begin{bmatrix}0 & amp;0 & amp;-3\\9 & amp;-1 & amp;5\\2 & amp;1 & amp;1\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}4 & amp;1 & amp;-1\\9 & amp;2 & amp;7\\3 & amp;-1 & amp;4\end{bmatrix}-\begin{bmatrix}4 & amp;1 & amp;2\\0 & amp;3 & amp;2\\1 & amp;-2 & amp;3\end{bmatrix}$$

$$=\begin{bmatrix}4-4 & amp;1-1 & amp;-1-2\\9-0 & amp;2-3 & amp;7-2\\3-1 & amp;-1-(-2) & amp;4-3\end{bmatrix}$$

$$=\begin{bmatrix}0 & amp;0 & amp;-3\\9 & amp;-1 & amp;5\\2 & amp;1 & amp;1\end{bmatrix}$$

Hence, we have verified that $A+(B-C)=(A+B)-C$.

14. If$A=\begin{bmatrix}\frac{2}{3} & amp;1 & amp;\frac{5}{3}\\\frac{1}{3} & amp;\frac{2}{3} & amp;\frac{4}{3}\\\frac{7}{3} & amp;2 & amp;\frac{2}{3}\end{bmatrix} andB=\begin{bmatrix}\frac{2}{5} & amp;\frac{3}{5} & amp;1\\\frac{1}{5} & amp;\frac{2}{5} & amp;\frac{4}{5}\\\frac{7}{5} & amp;\frac{6}{5} & amp;\frac{2}{5}\end{bmatrix}$ then compute $3A-5B$

Solutions :

$$3A-5B=3\begin{bmatrix}\frac{2}{3} & amp;1 & amp;\frac{5}{3}\\\frac{1}{3} & amp;\frac{2}{3} & amp;\frac{4}{3}\\\frac{7}{3} & amp;2 & amp;\frac{2}{3}\end{bmatrix}-5\begin{bmatrix}\frac{2}{5} & amp;\frac{3}{5} & amp;1\\\frac{1}{5} & amp;\frac{2}{5} & amp;\frac{4}{5}\\\frac{7}{5} & amp;\frac{6}{5} & amp;\frac{2}{5}\end{bmatrix}$$

$$=\begin{bmatrix}2 & amp;3 & amp;5\\1 & amp;2 & amp;4\\7 & amp;6 & amp;2\end{bmatrix}-\begin{bmatrix}2 & amp;3 & amp;5\\1 & amp;2 & amp;4\\7 & amp;6 & amp;2\end{bmatrix}$$

$$=\begin{bmatrix}0 & amp;0 & amp;0\\0 & amp;0 & amp;0\\0 & amp;0 & amp;0\end{bmatrix}$$

15. Simplify $\cos\theta\begin{bmatrix}\cos\theta & amp;\sin\theta\\-\sin\theta & amp;\cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & amp;-\cos\theta\\\cos\theta & amp;\sin\theta\end{bmatrix}$

Solutions :

$$\cos\theta\begin{bmatrix}\cos\theta & amp;\sin\theta\\-\sin\theta & amp;\cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & amp;-\cos\theta\\\cos\theta & amp;\sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta & amp;\cos\theta\sin\theta\\-\sin\theta\cos\theta & amp;\cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & amp;-\sin\theta\cos\theta\\\sin\theta\cos\theta & amp;\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & amp;\cos\theta\sin\theta-\sin\theta\cos\theta\\-\sin\theta\cos\theta+\sin\theta\cos\theta & amp;\cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & amp;0\\0 & amp;1\end{bmatrix}\qquad(\therefore\sin^2\theta=1)$$

16. Find X and Y ,if

$(i)X+Y=\begin{bmatrix}7 & amp;0\\2 & amp;5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & amp;0\\0 & amp;3\end{bmatrix}$

$(ii)2X+3Y=\begin{bmatrix}2 & amp;3\\4 & amp;0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & amp;-2\\-1 & amp;5\end{bmatrix}$

Solutions :

$(i)X+Y=\begin{bmatrix}7 & amp;0\\2 & amp;5\end{bmatrix}$ ......(1) $X-Y=\begin{bmatrix}3 & amp;0\\0 & amp;3\end{bmatrix}$.......(2)

Adding equations(1) and (2) , we get:

$$2X=\begin{bmatrix}7 & amp;0\\2 & amp;5\end{bmatrix}+X-Y=\begin{bmatrix}3 & amp;0\\0 & amp;3\end{bmatrix}$$

$$\begin{bmatrix}7+3 & amp;0+0\\2+0 & amp;5+3\end{bmatrix}=\begin{bmatrix}10 & amp;0\\2 & amp;8\end{bmatrix}$$

$$\therefore X=\frac{1}{2}\begin{bmatrix}10 & amp;0\\2 & amp;8\end{bmatrix}=\begin{bmatrix}5 & amp;0\\1 & amp;4\end{bmatrix}$$

Now, $X+Y=\begin{bmatrix}7 & amp;0\\2 & amp;5\end{bmatrix}$

$$\Longrightarrow\begin{bmatrix}5 & amp;0\\1 & amp;4\end{bmatrix}+Y=\begin{bmatrix}7 & amp;0\\2 & amp;5\end{bmatrix}$$

$$Y=\begin{bmatrix}7 & amp;0\\2 & amp;5\end{bmatrix}-\begin{bmatrix}5 & amp;0\\1 & amp;4\end{bmatrix}$$

$$\implies Y = \begin{bmatrix} 7-5 \quad amp;0-0 \\ 2-1 \quad amp;5-4 \end{bmatrix}$$
$$\therefore Y = \begin{bmatrix} 2 \quad amp;0 \\ 1 \quad amp;1 \end{bmatrix}$$
$$\implies 2\begin{bmatrix} \frac{2}{5} \quad amp;-\frac{12}{5} \\ -\frac{11}{5} \quad amp;3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 \quad amp;3 \\ 4 \quad amp;0 \end{bmatrix}$$
$$\implies \begin{bmatrix} \frac{4}{5} \quad amp;-\frac{24}{5} \\ -\frac{22}{5} \quad amp;6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 \quad amp;3 \\ 4 \quad amp;0 \end{bmatrix}$$
$$\implies 3Y = \begin{bmatrix} 2 \quad amp;3 \\ 4 \quad amp;0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} \quad amp;-\frac{24}{5} \\ -\frac{22}{5} \quad amp;6 \end{bmatrix}$$
$$3Y = \begin{bmatrix} 2-\frac{4}{5} \quad amp;3+\frac{24}{5} \\ 4+\frac{22}{5} \quad amp;0-6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} \quad amp;\frac{39}{5} \\ \frac{42}{5} \quad amp;-6 \end{bmatrix}$$
$$\therefore Y = \frac{1}{3}\begin{bmatrix} \frac{6}{5} \quad amp;\frac{13}{5} \\ \frac{42}{5} \quad amp;-6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \quad amp;\frac{13}{5} \\ \frac{14}{5} \quad amp;-2 \end{bmatrix}$$

$(ii) 2X + 3Y = \begin{bmatrix} 2 \quad amp;3 \\ 4 \quad amp;0 \end{bmatrix}$.......(3) $3X + 2Y = \begin{bmatrix} 2 \quad amp;-2 \\ -1 \quad amp;5 \end{bmatrix}$.....(4) Multiplying equation (3) with (2), we get

$$2(2X+3Y) = 2\begin{bmatrix} 2 \quad amp;3 \\ 4 \quad amp;0 \end{bmatrix}$$
$$\implies 4X + 6Y = \begin{bmatrix} 4 \quad amp;6 \\ 8 \quad amp;0 \end{bmatrix} \text{....(5)}$$

Multiplying equation (4) with (3), we get

$$3(3X+2Y) = 3\begin{bmatrix} 2 \quad amp;-2 \\ -1 \quad amp;5 \end{bmatrix}$$
$$\implies 9X + 6Y = \begin{bmatrix} 6 \quad amp;-6 \\ -3 \quad amp;15 \end{bmatrix} \text{.....(6)}$$

From(5) and (6),we have

$$(4X+6Y) - (9X+6Y) = \begin{bmatrix} 4 \quad amp;6 \\ 8 \quad amp;0 \end{bmatrix} - \begin{bmatrix} 6 \quad amp;-6 \\ -3 \quad amp;15 \end{bmatrix}$$
$$\implies -5X = \begin{bmatrix} 4-6 \quad amp;6-(-6) \\ 8-(-3) \quad amp;0-15 \end{bmatrix} = \begin{bmatrix} -2 \quad amp;12 \\ 11 \quad amp;-15 \end{bmatrix}$$
$$\therefore X = -\frac{1}{5}\begin{bmatrix} -2 \quad amp;12 \\ 11 \quad amp;-15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \quad amp;-\frac{12}{5} \\ -\frac{11}{5} \quad amp;3 \end{bmatrix}$$
$$2X + 3Y = \begin{bmatrix} 2 \quad amp;3 \\ 4 \quad amp;0 \end{bmatrix}$$

Now ,

17. Find X,if $Y = \begin{bmatrix} 3 \quad amp;2 \\ 1 \quad amp;4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 \quad amp;0 \\ -3 \quad amp;2 \end{bmatrix}$

Solutions :

$$2X + Y = \begin{bmatrix} 1 \quad amp;0 \\ -3 \quad amp;2 \end{bmatrix}$$
$$\implies 2X + \begin{bmatrix} 3 \quad amp;2 \\ 1 \quad amp;4 \end{bmatrix} = \begin{bmatrix} 1 \quad amp;0 \\ -3 \quad amp;2 \end{bmatrix}$$
$$\implies 2X = \begin{bmatrix} 1 \quad amp;0 \\ -3 \quad amp;2 \end{bmatrix} - \begin{bmatrix} 3 \quad amp;2 \\ 1 \quad amp;4 \end{bmatrix} = \begin{bmatrix} 1-3 \quad amp;0-2 \\ -3-1 \quad amp;2-4 \end{bmatrix}$$
$$\implies 2X = \begin{bmatrix} -2 \quad amp;-2 \\ -4 \quad amp;-2 \end{bmatrix}$$
$$\therefore X = \frac{1}{2}\begin{bmatrix} -2 \quad amp;-2 \\ -4 \quad amp;-2 \end{bmatrix} = \begin{bmatrix} -1 \quad amp;-1 \\ -2 \quad amp;-1 \end{bmatrix}$$

18. Find X and Y,if $2\begin{bmatrix} 1 \quad amp;3 \\ 0 \quad amp;x \end{bmatrix} + \begin{bmatrix} y \quad amp;0 \\ 1 \quad amp;2 \end{bmatrix} = \begin{bmatrix} 5 \quad amp;6 \\ 1 \quad amp;8 \end{bmatrix}$

Solutions :

$$2\begin{bmatrix} 1 \quad amp;3 \\ 0 \quad amp;x \end{bmatrix} + \begin{bmatrix} y \quad amp;0 \\ 1 \quad amp;2 \end{bmatrix} = \begin{bmatrix} 5 \quad amp;6 \\ 1 \quad amp;8 \end{bmatrix}$$
$$\implies \begin{bmatrix} 2 \quad amp;6 \\ 0 \quad amp;2x \end{bmatrix} + \begin{bmatrix} y \quad amp;0 \\ 1 \quad amp;2 \end{bmatrix} = \begin{bmatrix} 5 \quad amp;6 \\ 1 \quad amp;8 \end{bmatrix}$$
$$\begin{bmatrix} 2+y \quad amp;6 \\ 1 \quad amp;2x+2 \end{bmatrix} = \begin{bmatrix} 5 \quad amp;6 \\ 1 \quad amp;8 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we have:

$2 + y = 5 \implies y = 3$

$2x+2=8 \implies x=3$
$\therefore x=3 \text{and } y=3$

19. Solve the equation for x, y, z and t if
$2\begin{bmatrix} x & amp; z \\ y & amp; t \end{bmatrix} + 3\begin{bmatrix} 1 & amp; -1 \\ 0 & amp; 2 \end{bmatrix} = 3\begin{bmatrix} 3 & amp; 5 \\ 4 & amp; 6 \end{bmatrix}$

Solutions :

$2\begin{bmatrix} x & amp; z \\ y & amp; t \end{bmatrix} + 3\begin{bmatrix} 1 & amp; -1 \\ 0 & amp; 2 \end{bmatrix} = 3\begin{bmatrix} 3 & amp; 5 \\ 4 & amp; 6 \end{bmatrix}$
$\implies \begin{bmatrix} 2x & amp; 2z \\ 2y & amp; 2t \end{bmatrix} + \begin{bmatrix} 3 & amp; -3 \\ 0 & amp; 6 \end{bmatrix} = \begin{bmatrix} 9 & amp; 15 \\ 12 & amp; 18 \end{bmatrix}$
$\implies \begin{bmatrix} 2x+3 & amp; 2z-3 \\ 2y & amp; 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & amp; 15 \\ 12 & amp; 18 \end{bmatrix}$
Comparing the corresponding elements of these two matrices, we get:
$2x+3=9$
$\implies 2x=6$
$\implies x=3$

2 y = 12
$\implies y=6$

2 z - 3 = 15
$\implies 2z=18$
$\implies z=9$

$2t+6=18$
$\implies 2t=12 \implies t=6 \therefore x=3, y=6, z=9, \text{and } \ t=6$

20. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find values of x and y

Solutions :

$x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$
$\implies \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$
$\implies \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} + \begin{bmatrix} 10 \\ 5 \end{bmatrix}$
Comparing the corresponding elements of these two matrices, we get:
$2x-y=10$ and $3x+y=5$
Adding these two equations, we have:
$5x=15 \implies x=3$
Now, $3x+y=5$
$\implies y=5-3x$
$\implies y=5-9=-4$
$\therefore x=3 \text{and } y=-4$

21. Given $3\begin{bmatrix} x & amp; y \\ z & amp; w \end{bmatrix} = \begin{bmatrix} x & amp; 6 \\ -1 & amp; 2w \end{bmatrix} + \begin{bmatrix} 4 & amp; x+y \\ z+w & amp; 3 \end{bmatrix}$, find the values of x,y,z and w.

Solutions :

$3\begin{bmatrix} x & amp; y \\ z & amp; w \end{bmatrix} = \begin{bmatrix} x & amp; 6 \\ -1 & amp; 2w \end{bmatrix} + \begin{bmatrix} 4 & amp; x+y \\ z+w & amp; 3 \end{bmatrix}$,
$\implies \begin{bmatrix} 3x & amp; 3y \\ 3z & amp; 3w \end{bmatrix} = \begin{bmatrix} x+4 & amp; 6+x+y \\ -1+z+w & amp; 2w+3 \end{bmatrix}$
Comparing the corresponding elements of these two matrices, we get:
$3x=x+4$
$\implies 2x=4$
$\implies x=2$
$3x=6+x+y$
$\implies 2y=6+x=6+2=8$
$\implies y=4$
$3w=2w+3 \implies w=3$

$3z=-1+z+w$
$\implies 2z=-1+w=-1+3=2 \implies z=1$
$\therefore x=2, y=4, z=1, \text{and } \ w=3$

22. If$F(x) = \begin{bmatrix} \cos x & amp; -\sin x & amp; 0 \\ \sin x & amp; \cos x & amp; 0 \\ 0 & amp; 0 & amp; 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y.)$

Solutions :

F(x)= $\begin{bmatrix} \cos x & amp;-\sin x & amp;0 \\ \sin x & amp;\cos x & amp;0 \\ 0 & amp;0 & amp;1 \end{bmatrix}, F(x) = \begin{bmatrix} \cos y & amp;-\sin y & amp;0 \\ \sin y & amp;\cos y & amp;0 \\ 0 & amp;0 & amp;1 \end{bmatrix}$

$R.H.S : F(x+y) = \begin{bmatrix} \cos(x+y) & amp;-\sin(x+y) & amp;0 \\ \sin(x+y) & amp;\cos(x+y) & amp;0 \\ 0 & amp;0 & amp;1 \end{bmatrix}$

$L.H.S : F(x)F(y) = \begin{bmatrix} \cos x & amp;-\sin x & amp;0 \\ \sin x & amp;\cos x & amp;0 \\ 0 & amp;0 & amp;1 \end{bmatrix} \begin{bmatrix} \cos y & amp;-\sin y & amp;0 \\ \sin y & amp;\cos y & amp;0 \\ 0 & amp;0 & amp;1 \end{bmatrix}$

$= \begin{bmatrix} \cos x \cos y - \sin x \sin y + 0 & amp;-\cos x \sin y - \sin x \cos y + 0 & amp;0 \\ \sin x \cos y + \cos x \sin y + 0 & amp;-\sin x \sin y + \cos x \cos y + 0 & amp;0 \\ 0 & amp;0 & amp;1 \end{bmatrix}$

$= \begin{bmatrix} \cos(x+y) & amp;-\sin(x+y) & amp;0 \\ \sin(x+y) & amp;\cos(x+y) & amp;0 \\ 0 & amp;0 & amp;1 \end{bmatrix}$

$= F(x+y)$

$\therefore F(x)F(y) = F(x+y)$

23. Show that $(i) \begin{bmatrix} 5 & amp;-1 \\ 6 & amp;7 \end{bmatrix} \begin{bmatrix} 2 & amp;1 \\ 3 & amp;4 \end{bmatrix} \neq \begin{bmatrix} 2 & amp;1 \\ 3 & amp;4 \end{bmatrix} \begin{bmatrix} 5 & amp;-1 \\ 6 & amp;7 \end{bmatrix}$

$(ii) \begin{bmatrix} 1 & amp;2 & amp;3 \\ 0 & amp;1 & amp;0 \\ 1 & amp;1 & amp;0 \end{bmatrix} \begin{bmatrix} -1 & amp;1 & amp;0 \\ 0 & amp;-1 & amp;1 \\ 2 & amp;3 & amp;4 \end{bmatrix} \neq \begin{bmatrix} -1 & amp;1 & amp;0 \\ 0 & amp;-1 & amp;1 \\ 2 & amp;3 & amp;4 \end{bmatrix} \begin{bmatrix} 1 & amp;2 & amp;3 \\ 0 & amp;1 & amp;0 \\ 1 & amp;1 & amp;0 \end{bmatrix}$

Solutions :

$\begin{bmatrix} 5 & amp;-1 \\ 6 & amp;7 \end{bmatrix} \begin{bmatrix} 2 & amp;1 \\ 3 & amp;4 \end{bmatrix}$

$\begin{bmatrix} 5(2)-1(3) & amp;5(1)-1(4) \\ 6(2)+7(3) & amp;6(1)+7(4) \end{bmatrix}$

$\begin{bmatrix} 10-3 & amp;5-4 \\ 12+21 & amp;6+28 \end{bmatrix} = \begin{bmatrix} 7 & amp;1 \\ 33 & amp;34 \end{bmatrix}$

$\begin{bmatrix} 2 & amp;1 \\ 3 & amp;4 \end{bmatrix} \begin{bmatrix} 5 & amp;-1 \\ 6 & amp;7 \end{bmatrix}$

$= \begin{bmatrix} 2(5)+1(6) & amp;2(-1)+1(7) \\ 3(5)+4(6) & amp;3(-1)+4(7) \end{bmatrix}$

$= \begin{bmatrix} 10+6 & amp;-2+7 \\ 15+24 & amp;-3+28 \end{bmatrix} = \begin{bmatrix} 16 & amp;5 \\ 39 & amp;25 \end{bmatrix}$

$\therefore \begin{bmatrix} 5 & amp;-1 \\ 6 & amp;7 \end{bmatrix} \begin{bmatrix} 2 & amp;1 \\ 3 & amp;4 \end{bmatrix} \neq \begin{bmatrix} 2 & amp;1 \\ 3 & amp;4 \end{bmatrix} \begin{bmatrix} 5 & amp;-1 \\ 6 & amp;7 \end{bmatrix}$

$(ii) \begin{bmatrix} 1 & amp;2 & amp;3 \\ 0 & amp;1 & amp;0 \\ 1 & amp;1 & amp;0 \end{bmatrix} \begin{bmatrix} -1 & amp;1 & amp;0 \\ 0 & amp;-1 & amp;1 \\ 2 & amp;3 & amp;4 \end{bmatrix}$

$\begin{bmatrix} 1(-1)+2(0)+3(2) & amp;1(1)+2(-1)+3(3) & amp;1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & amp;0(1)+1(-1)+0(3) & amp;0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & amp;1(1)+1(-1)+0(3) & amp;1(0)+1(1)+0(4) \end{bmatrix}$

$\begin{bmatrix} 5 & amp;8 & amp;14 \\ 0 & amp;-1 & amp;1 \\ -1 & amp;0 & amp;1 \end{bmatrix}$

$\begin{bmatrix} -1 & amp;1 & amp;0 \\ 0 & amp;-1 & amp;1 \\ 2 & amp;3 & amp;4 \end{bmatrix} \begin{bmatrix} 1 & amp;2 & amp;3 \\ 0 & amp;1 & amp;0 \\ 1 & amp;1 & amp;0 \end{bmatrix}$

$\begin{bmatrix} -1(1)+1(0)+0(1) & amp;-1(2)+1(-1)+0(1) & amp;-1(3)+1(0)+0(0) \\ 0(1)+(-1)(0)+1(1) & amp;0(2)+(-1)(1)+1(1) & amp;0(3)+(-1)(0)+1(0) \\ 2(1)+3(0)+4(1) & amp;2(2)+3(1)+4(1) & amp;2(3)+3(0)+4(0) \end{bmatrix}$

$\begin{bmatrix} -1 & amp;-1 & amp;-3 \\ 1 & amp;0 & amp;0 \\ 6 & amp;11 & amp;6 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 & amp;2 & amp;3 \\ 0 & amp;1 & amp;0 \\ 1 & amp;1 & amp;0 \end{bmatrix} \begin{bmatrix} -1 & amp;1 & amp;0 \\ 0 & amp;-1 & amp;1 \\ 2 & amp;3 & amp;4 \end{bmatrix} \neq \begin{bmatrix} -1 & amp;1 & amp;0 \\ 0 & amp;-1 & amp;1 \\ 2 & amp;3 & amp;4 \end{bmatrix} \begin{bmatrix} 1 & amp;2 & amp;3 \\ 0 & amp;1 & amp;0 \\ 1 & amp;1 & amp;0 \end{bmatrix}$

24. Find $A^2 - 5A + 6I$ if $A = \begin{bmatrix} 2 & amp;0 & amp;1 \\ 2 & amp;1 & amp;3 \\ 1 & amp;-1 & amp;0 \end{bmatrix}$

Solutions :

We have $A^2 = A * A$

$$A^2 = AA \begin{bmatrix} 2 & amp;0 & amp;1 \\ 2 & amp;1 & amp;3 \\ 1 & amp;-1 & amp;0 \end{bmatrix} \begin{bmatrix} 2 & amp;0 & amp;1 \\ 2 & amp;1 & amp;3 \\ 1 & amp;-1 & amp;0 \end{bmatrix}$$

$$\begin{bmatrix} 2(2)+0(2)+1(1) & amp;2(0)+0(1)+1(-1) & amp;2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & amp;2(0)+1(1)+3(-1) & amp;2(1)+1(3)+3(0) \\ 1(2)+(-1)(2)+0(1) & amp;1(0)+(-1)(1)+0(-1) & amp;1(1)+(-1)(3)+0(0) \end{bmatrix}$$

$$\begin{bmatrix} 4+0+1 & amp;0+0-1 & amp;2+0+0 \\ 4+2+3 & amp;0+1-3 & amp;2+3+0 \\ 2-2+0 & amp;0-1+0 & amp;1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & amp;-1 & amp;2 \\ 9 & amp;-2 & amp;5 \\ 0 & amp;-1 & amp;-2 \end{bmatrix}$$

Substituting the matrices in the given equation :$A^2 - 5A + 6I$

$$= \begin{bmatrix} 5 & amp;-1 & amp;2 \\ 9 & amp;-2 & amp;5 \\ 0 & amp;-1 & amp;-2 \end{bmatrix} - 5 \begin{bmatrix} 2 & amp;0 & amp;1 \\ 2 & amp;1 & amp;3 \\ 1 & amp;-1 & amp;0 \end{bmatrix} + 6 \begin{bmatrix} 1 & amp;0 & amp;0 \\ 0 & amp;1 & amp;0 \\ 0 & amp;0 & amp;1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & amp;-1 & amp;2 \\ 9 & amp;-2 & amp;5 \\ 0 & amp;-1 & amp;-2 \end{bmatrix} - \begin{bmatrix} 10 & amp;0 & amp;5 \\ 10 & amp;5 & amp;15 \\ 5 & amp;-5 & amp;0 \end{bmatrix} + \begin{bmatrix} 6 & amp;0 & amp;0 \\ 0 & amp;6 & amp;0 \\ 0 & amp;0 & amp;6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10 & amp;-1-0 & amp;2-5 \\ 9-10 & amp;-2-5 & amp;5-15 \\ 0-5 & amp;-1+5 & amp;-2-0 \end{bmatrix} + \begin{bmatrix} 6 & amp;0 & amp;0 \\ 0 & amp;6 & amp;0 \\ 0 & amp;0 & amp;6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & amp;-1 & amp;-3 \\ -1 & amp;-7 & amp;-10 \\ -5 & amp;4 & amp;-2 \end{bmatrix} + \begin{bmatrix} 6 & amp;0 & amp;0 \\ 0 & amp;6 & amp;0 \\ 0 & amp;0 & amp;6 \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 & amp;-1+0 & amp;-3+0 \\ -1+0 & amp;-7+6 & amp;-10+0 \\ -5+0 & amp;4+0 & amp;-2+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & amp;-1 & amp;-3 \\ -1 & amp;-1 & amp;-10 \\ -5 & amp;4 & amp;4 \end{bmatrix}$$

25. If $A = \begin{bmatrix} 1 & amp;0 & amp;2 \\ 0 & amp;2 & amp;1 \\ 2 & amp;0 & amp;3 \end{bmatrix}$, prove that $A^3 - 6A^2 - 6A^2 + 7A + 2I = 0$

Solutions :

$$A^2 = AA = \begin{bmatrix} 1 & amp;0 & amp;2 \\ 0 & amp;2 & amp;1 \\ 2 & amp;0 & amp;3 \end{bmatrix} \begin{bmatrix} 1 & amp;0 & amp;2 \\ 0 & amp;2 & amp;1 \\ 2 & amp;0 & amp;3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & amp;0+0+0 & amp;2+0+6 \\ 0+0+2 & amp;0+4+0 & amp;0+2+3 \\ 2+0+6 & amp;0+0+0 & amp;4+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & amp;0 & amp;8 \\ 2 & amp;4 & amp;5 \\ 8 & amp;0 & amp;13 \end{bmatrix}$$

Now, $A^3 = A^2.A$

$$\begin{bmatrix} 5 & amp;0 & amp;8 \\ 2 & amp;4 & amp;5 \\ 8 & amp;0 & amp;13 \end{bmatrix} \begin{bmatrix} 1 & amp;0 & amp;2 \\ 0 & amp;2 & amp;1 \\ 2 & amp;0 & amp;3 \end{bmatrix}$$

$$\begin{bmatrix} 5+0+16 & amp;0+0+0 & amp;10+0+24 \\ 2+0+10 & amp;0+8+0 & amp;4+4+15 \\ 8+0+26 & amp;0+0+0 & amp;16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & amp;0 & amp;34 \\ 12 & amp;8 & amp;23 \\ 34 & amp;0 & amp;55 \end{bmatrix}$$

Substituting the matrices in the given equation $A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & amp;0 & amp;34 \\ 12 & amp;8 & amp;23 \\ 34 & amp;0 & amp;55 \end{bmatrix} - 6 \begin{bmatrix} 5 & amp;0 & amp;8 \\ 2 & amp;4 & amp;5 \\ 8 & amp;0 & amp;13 \end{bmatrix} + \begin{bmatrix} 1 & amp;0 & amp;2 \\ 0 & amp;2 & amp;1 \\ 2 & amp;0 & amp;3 \end{bmatrix} + 2 \begin{bmatrix} 1 & amp;0 & amp;0 \\ 0 & amp;1 & amp;0 \\ 0 & amp;0 & amp;1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & amp;0 & amp;34 \\ 12 & amp;8 & amp;23 \\ 34 & amp;0 & amp;55 \end{bmatrix} - \begin{bmatrix} 30 & amp;0 & amp;48 \\ 12 & amp;24 & amp;30 \\ 48 & amp;0 & amp;78 \end{bmatrix} + \begin{bmatrix} 7 & amp;0 & amp;14 \\ 0 & amp;14 & amp;7 \\ 14 & amp;0 & amp;21 \end{bmatrix} + \begin{bmatrix} 2 & amp;0 & amp;0 \\ 0 & amp;2 & amp;0 \\ 0 & amp;0 & amp;2 \end{bmatrix}$$

$$= \begin{bmatrix} 21+7+2 & amp;0+0+0 & amp;34+14+0 \\ 12+0+0 & amp;8+14+2 & amp;23+7+0 \\ 34+14+0 & amp;0+0+0 & amp;55+21+2 \end{bmatrix} - \begin{bmatrix} 30 & amp;0 & amp;48 \\ 12 & amp;24 & amp;30 \\ 48 & amp;0 & amp;78 \end{bmatrix}$$

$= \begin{bmatrix} 30 & amp;0 & amp;48 \\ 12 & amp;24 & amp;30 \\ 48 & amp;0 & amp;78 \end{bmatrix} - \begin{bmatrix} 30 & amp;0 & amp;48 \\ 12 & amp;24 & amp;30 \\ 48 & amp;0 & amp;78 \end{bmatrix}$

$= \begin{bmatrix} 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \end{bmatrix} = 0$

$\therefore A^3 - 6A^2 + 7A + 2I = 0$

26. If $A = \begin{bmatrix} 3 & amp;-2 \\ 4 & amp;-2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & amp;0 \\ 0 & amp;1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

Solutions :

$A^2 = A.A = \begin{bmatrix} 3 & amp;-2 \\ 4 & amp;-2 \end{bmatrix} \begin{bmatrix} 3 & amp;-2 \\ 4 & amp;-2 \end{bmatrix}$

$= \begin{bmatrix} 3(3)+(-2)(4) & amp;3(-2)+(-2)(-2) \\ 4(3)+(-2)(4) & amp;4(-2)+(-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & amp;-2 \\ 4 & amp;-4 \end{bmatrix}$

Now $A^2 = kA - 2I$

$\implies \begin{bmatrix} 1 & amp;-2 \\ 4 & amp;-4 \end{bmatrix} = k \begin{bmatrix} 3 & amp;-2 \\ 4 & amp;-2 \end{bmatrix} - 2 \begin{bmatrix} 1 & amp;0 \\ 0 & amp;1 \end{bmatrix}$

$\implies \begin{bmatrix} 1 & amp;-2 \\ 4 & amp;-4 \end{bmatrix} = \begin{bmatrix} 3k & amp;-2k \\ 4k & amp;-2k \end{bmatrix} - \begin{bmatrix} 2 & amp;0 \\ 0 & amp;2 \end{bmatrix}$

$= \begin{bmatrix} 1 & amp;-2 \\ 4 & amp;-4 \end{bmatrix} = \begin{bmatrix} 3k-2 & amp;-2k \\ 4k & amp;-2k-2 \end{bmatrix}$

Comparing the corresponding elements, we have:

$3k - 2 = 1$

$\implies 3k = 3$

$\implies k = 1$ Thus, the value of k is 1.

27. If $A = \begin{bmatrix} 0 & amp;-\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & amp;0 \end{bmatrix}$ and I is the identity matrix of order 2,show that $I + A = (I - A) \begin{bmatrix} \cos\alpha & amp;-\sin\alpha \\ \sin\alpha & amp;\cos\alpha \end{bmatrix}$

Solutions :

$= \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & amp;-2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + (2\cos^2\frac{alpha}{2} - 1)\tan\frac{\alpha}{2} \\ -(2\cos^2\frac{\alpha}{2} - 1)\tan\frac{alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & amp;2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin^2\frac{\alpha}{2} & amp;-2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & amp;2\sin^2\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 & amp;-\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & amp;1 \end{bmatrix}$

Thus, from (1) and (2), we get $L.H.S. = R.H.S.$

$= \left(\begin{bmatrix} 1 & amp;0 \\ 0 & amp;1 \end{bmatrix} - \begin{bmatrix} 0 & amp;-\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & amp;0 \end{bmatrix} \right) \begin{bmatrix} \cos\alpha & amp;-\sin\alpha \\ \sin\alpha & amp;\cos\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & amp;\tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & amp;1 \end{bmatrix} \begin{bmatrix} \cos\alpha & amp;-\sin\alpha \\ \sin\alpha & amp;\cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & amp;-\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & amp;\sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix} ....(2)$

$= \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin^2\frac{\alpha}{2} & amp;-2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & amp;2\sin^2\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 & amp;-\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & amp;1 \end{bmatrix}$

Thus, from (1) and (2), we get $L.H.S. = R.H.S.$

$L.H.S.$

$I + A$

$= \begin{bmatrix} 1 & amp;0 \\ 0 & amp;1 \end{bmatrix} + \begin{bmatrix} 0 & amp;-\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & amp;0 \end{bmatrix}$

$\begin{bmatrix} 1 & amp;-\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & amp;1 \end{bmatrix} ....(1)$

$R.H.S.$

$(I - A) \begin{bmatrix} \cos\alpha & amp;-\sin\alpha \\ \sin\alpha & amp;\cos\alpha \end{bmatrix}$

$= \left(\begin{bmatrix} 1 & amp;0 \\ 0 & amp;1 \end{bmatrix} - \begin{bmatrix} 0 & amp;-\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & amp;0 \end{bmatrix} \right) \begin{bmatrix} \cos\alpha & amp;-\sin\alpha \\ \sin\alpha & amp;\cos\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & amp; \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & amp; 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & amp; -\sin\alpha \\ \sin\alpha & amp; \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & amp; -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & amp; \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$ ....(2)

$= \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & amp; -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + (2\cos^2\frac{alpha}{2} - 1)\tan\frac{\alpha}{2} \\ -(2\cos^2\frac{\alpha}{2} - 1)\tan\frac{alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & amp; 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 - 2\sin^2\frac{\alpha}{2} + 2\sin^2\frac{\alpha}{2} & amp; -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & amp; 2\sin^2\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 & amp; -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & amp; 1 \end{bmatrix}$

Thus, from (1) and (2), we get $L.H.S. = R.H.S.$

28. A trust fund has $Rs30,000$ that must be invested in two different types of bonds.
The first bond pays 5 Using matrix multiplication, determine how to divide $Rs30,000$ among the two types of bonds.
If the trust fund must obtain an annual total interest of : (a)$Rs1,800$
(b) $Rs2,000$

Solutions :

(b) Let Rs x be invested in the first bond. Then,
the sum of money invested in the second bond will be $Rs(30000 - x)$
. Therefore, in order to obtain an annual total interest of $Rs2000$,
we have :

$[x(30000 - x)] \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$

$\implies \frac{5x}{100} + \frac{7(30000 - x)}{100}$
$= 2000$
$\implies 5x + 210000 - 7x$
$= 200000$
$\implies 210000 - 2x = 200000$
$\implies 2x = 210000 - 200000$
$\implies 2x = 10000$
$\implies x = 5000$

Thus, in order to obtain an annual total interest of $Rs2000$,
the trust fund should invest $Rs5000$ in the first bond
and the remaining $Rs25000$ in the second bond.

a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second
bond pays $Rs(30000 - x)$.
It is given that the first bond pays 5year.
Therefore, in order to obtain an annual total interest of $Rs1800$,
we have:

$[x(30000 - x)] \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} [S.I.\text{for1year} = \frac{\text{Principal * Rate}}{100}]$

$\implies \frac{5x}{100} + \frac{7(30000 - x)}{100} = 1800$
$\implies 5x + 210000 - 7x = 180000$
$\implies 210000 - 2x = 180000$
$2x = 210000 - 180000$
$2x = 30000$
$x = 15000$

Thus, in order to obtain an annual total interest of $Rs1800$,
the trust fund should invest $Rs15000$ in the first bond
and the remaining $Rs15000$ in the second bond.

(b) Let Rs x be invested in the first bond. Then,
the sum of money invested in the second bond will be $Rs(30000 - x)$
. Therefore, in order to obtain an annual total interest of $Rs2000$,
we have :

$[x(30000 - x)] \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$

$\implies \frac{5x}{100} + \frac{7(30000 - x)}{100}$
$= 2000$
$\implies 5x + 210000 - 7x$
$= 200000$

$\implies 210000 - 2x = 200000$
$\implies 2x = 210000 - 200000$
$\implies 2x = 10000$
$\implies x = 5000$
Thus, in order to obtain an annual total interest of $Rs2000$,
the trust fund should invest $Rs5000$ in the first bond
and the remaining $Rs25000$ in the second bond.

29. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are $Rs80, Rs60$ and $Rs40$ each respectively.
Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Solutions :

The bookshop has 10 dozen chemistry books,
8 dozen physics books, and 10 dozen economics books.
The selling prices of a chemistry book, a physics book,
and an economics book are respectively given as $Rs80, Rs60$ and $Rs40$.
The total amount of money that will be received from the
sale of all these books can be represented in the form of a matrix as :

$12\begin{bmatrix}10 & amp; 8 & amp; 10\end{bmatrix}\begin{bmatrix}80\\60\\40\end{bmatrix}$
$= 12[10 * 80 + 8 * 60 + 10 * 40]$
$= 12(800 + 480 + 400)$
$= 12(1680)$
20160
Thus, the bookshop will receive $Rs20160$ from the sale of all these books.

30. Assume X, Y, Z, W and P are the matrices of order
$2 * n, 3 * k, 2 * p, n * 3$ and $p * k$ respectively.
The restriction on n, k and p so that $PY + WY$ will be defined are:
A.$k = 3, p = n$
B. k is arbitrary, $p = 2$
C. p is arbitrary, $k = 3$
D.$k = 2, p = 3$

Solutions :

Matrices P and Y are of the orders
$p * k$ and $3 * k$ respectively.
Therefore, matrix PY will be defined if $k * 3$.
Consequently, PY will be of the order $p * k$.
Matrices W and Y are of the orders
$n * 3$ and $3 * k$ respectively.
Since the number of columns in W is equal to the number of rows in Y ,
matrix WY is well- defined and is of the order $n * k$.
Matrices PY and WY can be added only when their orders are the same.
However,PY is of the order $p * k$
and WY is of the order $n * k$.
Therefore. we must have $p * n$.
Thus, $k * 3$ and $p * n$. are the restrictions on n, k, and p
so that $PY + WY$ will be defined

31. Assume X, Y, Z, W and P are matrices of order $2 * n, 3 * k, 2 * p, n * 3$ and $p * k$ respectively.
If $n * p$, then the order of the matrix $7X - 5Z$ is
$A.p * 2$
$B.2 * n$
$C.n * 3$
$D.p * n$

Solutions :

The correct answer is B.
Matrix X is of the order $2 * n$.
Therefore, matrix $7X$ is also of the same order.
Matrix Z is of the order $2 * p$, i.e, $2 * n$

[since $n * p$]
Therefore, matrix $5Z$ is also of the same order.
Now, both the matrices $7X$ and $5Z$ are of the order $2 * n$.
Thus, matrix $7X - 5Z$ is well-defined and is of the order $2 * n$.

32. Find the transpose of each of the following matrices:
(i) $\begin{bmatrix}5\\\frac{1}{2}\\-1\end{bmatrix}$
(ii) $\begin{bmatrix}1 & amp; -1\\2 & amp; 3\end{bmatrix}$

(iii) $\begin{bmatrix} -1 & amp;5 & amp;6 \\ \sqrt{3} & amp;5 & amp;6 \\ 2 & amp;3 & amp;-1 \end{bmatrix}$

Solutions :

(i)Let A= $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$, then

$A^T = \begin{bmatrix} 5 & amp;\frac{1}{2} & amp;-1 \end{bmatrix}$

(ii)Let A= $\begin{bmatrix} 1 & amp;-1 \\ 2 & amp;3 \end{bmatrix}$, then

$A^T = \begin{bmatrix} 1 & amp;2 \\ -1 & amp;3 \end{bmatrix}$

(iii)Let A= $\begin{bmatrix} -1 & amp;5 & amp;6 \\ \sqrt{3} & amp;5 & amp;6 \\ 2 & amp;3 & amp;-1 \end{bmatrix}$, then

$A^T = \begin{bmatrix} -1 & amp;\sqrt{3} & amp;2 \\ 5 & amp;5 & amp;3 \\ 6 & amp;6 & amp;-1 \end{bmatrix}$

33. If$A = \begin{bmatrix} -1 & amp;2 & amp;3 \\ 5 & amp;7 & amp;9 \\ -2 & amp;1 & amp;1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & amp;1 & amp;-5 \\ 1 & amp;2 & amp;0 \\ 1 & amp;3 & amp;1 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$
(ii)$(A-B)' = A' - B'$

Solutions :

We have:

$A' = \begin{bmatrix} -1 & amp;5 & amp;-2 \\ 2 & amp;7 & amp;1 \\ 3 & amp;9 & amp;1 \end{bmatrix}$, $B' = \begin{bmatrix} -4 & amp;1 & amp;1 \\ 1 & amp;2 & amp;3 \\ -5 & amp;0 & amp;1 \end{bmatrix}$

(i)$A + B = \begin{bmatrix} -1 & amp;2 & amp;3 \\ 5 & amp;7 & amp;9 \\ -2 & amp;1 & amp;1 \end{bmatrix} + \begin{bmatrix} -4 & amp;1 & amp;-5 \\ 1 & amp;2 & amp;0 \\ 1 & amp;3 & amp;1 \end{bmatrix}$

$= \begin{bmatrix} -5 & amp;3 & amp;-2 \\ 6 & amp;9 & amp;9 \\ -1 & amp;4 & amp;2 \end{bmatrix}$

$\therefore (A+B)' = \begin{bmatrix} -5 & amp;6 & amp;-1 \\ 3 & amp;9 & amp;4 \\ -2 & amp;9 & amp;2 \end{bmatrix}$

$A' + B' = \begin{bmatrix} -1 & amp;5 & amp;-2 \\ 2 & amp;7 & amp;1 \\ 3 & amp;9 & amp;1 \end{bmatrix} + \begin{bmatrix} -4 & amp;1 & amp;1 \\ 1 & amp;2 & amp;3 \\ -5 & amp;0 & amp;1 \end{bmatrix}$

$= \begin{bmatrix} -5 & amp;6 & amp;-1 \\ 3 & amp;9 & amp;4 \\ -2 & amp;9 & amp;2 \end{bmatrix}$

Hence, we have verified that$(A+B)' = A' + B'$

(ii)$A - B = \begin{bmatrix} -1 & amp;2 & amp;3 \\ 5 & amp;7 & amp;9 \\ -2 & amp;1 & amp;1 \end{bmatrix} - \begin{bmatrix} -4 & amp;1 & amp;-5 \\ 1 & amp;2 & amp;0 \\ 1 & amp;3 & amp;1 \end{bmatrix}$

$= \begin{bmatrix} 3 & amp;1 & amp;8 \\ 4 & amp;5 & amp;9 \\ -3 & amp;-2 & amp;0 \end{bmatrix}$

$\therefore (A-B)' = \begin{bmatrix} 3 & amp;4 & amp;-3 \\ 1 & amp;5 & amp;-2 \\ 8 & amp;9 & amp;0 \end{bmatrix}$

$A' - B' = \begin{bmatrix} -1 & amp;5 & amp;-2 \\ 2 & amp;7 & amp;1 \\ 3 & amp;9 & amp;1 \end{bmatrix} - \begin{bmatrix} -4 & amp;1 & amp;1 \\ 1 & amp;2 & amp;3 \\ -5 & amp;0 & amp;1 \end{bmatrix}$

$= \begin{bmatrix} 3 & amp;4 & amp;-3 \\ 1 & amp;5 & amp;-2 \\ 8 & amp;9 & amp;0 \end{bmatrix}$

Hence, we have verified that $(A-B)' = A' - B'$.

34. If $A' = \begin{bmatrix} 3 & amp;4 \\ -1 & amp;2 \\ 0 & amp;1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & amp;2 & amp;1 \\ 1 & amp;2 & amp;3 \end{bmatrix}$, then verify that

(i)$(A+B)' = A' + B'$
(ii)$(A-B)' = A' - B'$

Solutions :

(i) It is known that $A = (A')'$
therefore , we have:

$$A = \begin{bmatrix} 3 & amp; -1 & amp; 0 \\ 4 & amp; 2 & amp; 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & amp; 1 \\ 2 & amp; 2 \\ 1 & amp; 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & amp; -1 & amp; 0 \\ 4 & amp; 2 & amp; 1 \end{bmatrix} + \begin{bmatrix} -1 & amp; 2 & amp; 1 \\ 1 & amp; 2 & amp; 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & amp; 1 & amp; 1 \\ 5 & amp; 4 & amp; 4 \end{bmatrix}$$

$$\therefore (A+B)' = \begin{bmatrix} 2 & amp; 5 \\ 1 & amp; 4 \\ 1 & amp; 4 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} 3 & amp; 4 \\ -1 & amp; 2 \\ 0 & amp; 1 \end{bmatrix} + \begin{bmatrix} -1 & amp; 1 \\ 2 & amp; 2 \\ 1 & amp; 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & amp; 5 \\ 1 & amp; 4 \\ 1 & amp; 4 \end{bmatrix}$$

Thus, we verified that$(A+B)' = A' + B'$

$$\text{(ii)}A - B = \begin{bmatrix} 3 & amp; -1 & amp; 0 \\ 4 & amp; 2 & amp; 1 \end{bmatrix} - \begin{bmatrix} -1 & amp; 2 & amp; 1 \\ 1 & amp; 2 & amp; 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & amp; -3 & amp; -1 \\ 3 & amp; 0 & amp; -2 \end{bmatrix}$$

$$\therefore (A-B)' = \begin{bmatrix} 4 & amp; 3 \\ -3 & amp; 0 \\ -1 & amp; -2 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} 3 & amp; 4 \\ -1 & amp; 2 \\ 0 & amp; 1 \end{bmatrix} - \begin{bmatrix} -1 & amp; 1 \\ 2 & amp; 2 \\ 1 & amp; 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & amp; 3 \\ -3 & amp; 0 \\ -1 & amp; -2 \end{bmatrix}$$

Thus, we have verified that$(A-B)' = A' - B'$.

35. If $A' = \begin{bmatrix} -2 & amp; 3 \\ 1 & amp; 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & amp; 0 \\ 1 & amp; 2 \end{bmatrix}$,then find $(A+2B)$

Solutions :

We know that $A = (A')'$

$$\therefore A = \begin{bmatrix} -2 & amp; 1 \\ 3 & amp; 2 \end{bmatrix}$$

$$\therefore A + 2B = \begin{bmatrix} -2 & amp; 1 \\ 3 & amp; 2 \end{bmatrix} + 2\begin{bmatrix} -1 & amp; 0 \\ 1 & amp; 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & amp; 1 \\ 3 & amp; 2 \end{bmatrix} + \begin{bmatrix} -2 & amp; 0 \\ 2 & amp; 4 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & amp; 1 \\ 5 & amp; 6 \end{bmatrix}$$

$$\therefore (A+2B)' = \begin{bmatrix} -4 & amp; 5 \\ 1 & amp; 6 \end{bmatrix}$$

36. For the matrices A and B , verify that $(AB)' = B'A'$ where

$$\text{(i)}A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & amp; 2 & amp; 1 \end{bmatrix}$$

$$\text{(ii)}A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & amp; 5 & amp; 7 \end{bmatrix}$$

Solutions :

(i)$A=\begin{bmatrix}1\\-4\\3\end{bmatrix}\begin{bmatrix}-1 & amp;2 & amp;1\end{bmatrix}$

$=\begin{bmatrix}-1 & amp;2 & amp;1\\4 & amp;8 & amp;-4\\-3 & amp;6 & amp;3\end{bmatrix}$

$\therefore (AB)'=\begin{bmatrix}-1 & amp;4 & amp;-3\\2 & amp;-8 & amp;6\\1 & amp;-4 & amp;3\end{bmatrix}$

Now,$A=\begin{bmatrix}1 & amp;-4 & amp;3\end{bmatrix}, B=\begin{bmatrix}-1\\2\\1\end{bmatrix}$

$\therefore B'A'=\begin{bmatrix}-1\\2\\1\end{bmatrix}\begin{bmatrix}1 & amp;-4 & amp;3\end{bmatrix}$

$=\begin{bmatrix}-1 & amp;4 & amp;-3\\2 & amp;-8 & amp;6\\1 & amp;-4 & amp;3\end{bmatrix}$

Hence, we have verified $(AB)=B'A'$.

(ii)$AB=\begin{bmatrix}0\\1\\2\end{bmatrix}\begin{bmatrix}1 & amp;5 & amp;7\end{bmatrix}$

$=\begin{bmatrix}0 & amp;0 & amp;0\\1 & amp;5 & amp;7\\2 & amp;10 & amp;14\end{bmatrix}$

$\therefore (AB)'=\begin{bmatrix}0 & amp;1 & amp;2\\0 & amp;5 & amp;10\\0 & amp;7 & amp;14\end{bmatrix}$

Now , $A'=\begin{bmatrix}0 & amp;1 & amp;2\end{bmatrix}, B=\begin{bmatrix}1\\5\\7\end{bmatrix}$

$\therefore B'A'=\begin{bmatrix}1\\5\\7\end{bmatrix}\begin{bmatrix}0 & amp;1 & amp;2\end{bmatrix}$

$=\begin{bmatrix}0 & amp;1 & amp;2\\0 & amp;5 & amp;10\\0 & amp;7 & amp;14\end{bmatrix}$

Hence, we have verified that$(AB)'=B'A'$.

37. If (i) $A=\begin{bmatrix}\cos\alpha & amp;\sin\alpha\\-\sin\alpha & amp;\cos\alpha\end{bmatrix}$, then verify that $A'A=1$

(ii) $A=\begin{bmatrix}\sin\alpha & amp;\cos\alpha\\-\cos\alpha & amp;\sin\alpha\end{bmatrix}$, then verify that $A'A=1$

Solutions :

$A=\begin{bmatrix}\cos\alpha & amp;\sin\alpha\\-\sin\alpha & amp;\cos\alpha\end{bmatrix}$

$\therefore A'=\begin{bmatrix}\cos\alpha & amp;-\sin\alpha\\\sin\alpha & amp;\cos\alpha\end{bmatrix}$

$A'A=\begin{bmatrix}\cos\alpha & amp;-\sin\alpha\\\sin\alpha & amp;\cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & amp;-\sin\alpha\\-\sin\alpha & amp;\cos\alpha\end{bmatrix}$

$=\begin{bmatrix}(\cos\alpha)(\cos\alpha)+(-\sin\alpha)(-\sin\alpha) & amp;(\cos\alpha)(\cos\alpha)+(-\sin\alpha)(\cos\alpha)\\(\sin\alpha)(\cos\alpha)+(\cos\alpha)(-\sin\alpha) & amp;(\sin\alpha)(\sin\alpha)+(\cos\alpha)(\cos\alpha)\end{bmatrix}$

$\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & amp;\sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\\sin\alpha\cos\alpha-\sin\alpha\cos\alpha & amp;\sin^2\alpha+\cos^2\alpha\end{bmatrix}$

$=\begin{bmatrix}1 & amp;0\\0 & amp;1\end{bmatrix}=I$

Hence, we have verified that $A'A=1$.

(ii)$A=\begin{bmatrix}\sin\alpha & amp;\cos\alpha\\-\cos\alpha & amp;\sin\alpha\end{bmatrix}$

$\therefore A'=\begin{bmatrix}\sin\alpha & amp;-\cos\alpha\\\cos\alpha & amp;\sin\alpha\end{bmatrix}$

$A'A=\begin{bmatrix}\sin\alpha & amp;-\cos\alpha\\\cos\alpha & amp;\sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & amp;\cos\alpha\\-\cos\alpha & amp;\sin\alpha\end{bmatrix}$

$\begin{bmatrix}\sin\alpha & amp;-\cos\alpha\\\cos\alpha & amp;\sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & amp;\cos\alpha\\-\cos\alpha & amp;\sin\alpha\end{bmatrix}$

$= \begin{bmatrix} (\sin\alpha)(\sin\alpha)+(-\cos\alpha)(-\cos\alpha) & amp;(\sin\alpha)(\cos\alpha)+(-\cos\alpha)(\sin\alpha) \\ (\cos\alpha)(\sin\alpha)+(\sin\alpha)(-\cos\alpha) & amp;(\cos\alpha)(\cos\alpha)+(\sin\alpha)(\sin\alpha) \end{bmatrix}$

$= \begin{bmatrix} \sin^2\alpha+\cos^2\alpha & amp;\sin\alpha\cos\alpha-\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & amp;\cos^2\alpha+\sin^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & amp;0 \\ 0 & amp;1 \end{bmatrix} = I$

Hence, we have verified that $A'A = 1$.

$= \begin{bmatrix} (\cos\alpha)(\cos\alpha)+(-\sin\alpha)(-\sin\alpha) & amp;(\cos\alpha)(\cos\alpha)+(-\sin\alpha)(\cos\alpha) \\ (\sin\alpha)(\cos\alpha)+(\cos\alpha)(-\sin\alpha) & amp;(\sin\alpha)(\sin\alpha)+(\cos\alpha)(\cos\alpha) \end{bmatrix}$

$\begin{bmatrix} \cos^2\alpha+\sin^2\alpha & amp;\sin\alpha\cos\alpha-\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & amp;\sin^2\alpha+\cos^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & amp;0 \\ 0 & amp;1 \end{bmatrix} = I$

Hence, we have verified that $A'A = 1$.

$= \begin{bmatrix} (\sin\alpha)(\sin\alpha)+(-\cos\alpha)(-\cos\alpha) & amp;(\sin\alpha)(\cos\alpha)+(-\cos\alpha)(\sin\alpha) \\ (\cos\alpha)(\sin\alpha)+(\sin\alpha)(-\cos\alpha) & amp;(\cos\alpha)(\cos\alpha)+(\sin\alpha)(\sin\alpha) \end{bmatrix}$

$= \begin{bmatrix} \sin^2\alpha+\cos^2\alpha & amp;\sin\alpha\cos\alpha-\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & amp;\cos^2\alpha+\sin^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & amp;0 \\ 0 & amp;1 \end{bmatrix} = I$

Hence, we have verified that $A'A = 1$.

38. (i) Show that the matrix $A = \begin{bmatrix} 1 & amp;-1 & amp;5 \\ -1 & amp;2 & amp;1 \\ 5 & amp;1 & amp;3 \end{bmatrix}$ is a symmetric matrix

(ii) Show that the matrix$A = \begin{bmatrix} 0 & amp;1 & amp;-1 \\ -1 & amp;0 & amp;1 \\ 1 & amp;-1 & amp;0 \end{bmatrix}$ is a skew symmetric matrix

39. (i) Show that the matrix $A = \begin{bmatrix} 1 & amp;-1 & amp;5 \\ -1 & amp;2 & amp;1 \\ 5 & amp;1 & amp;3 \end{bmatrix}$ is a symmetric matrix

(ii) Show that the matrix$A = \begin{bmatrix} 0 & amp;1 & amp;-1 \\ -1 & amp;0 & amp;1 \\ 1 & amp;-1 & amp;0 \end{bmatrix}$ is a skew symmetric matrix

Solutions :

(i) We have :

$A' = \begin{bmatrix} 1 & amp;-1 & amp;5 \\ -1 & amp;2 & amp;1 \\ 5 & amp;1 & amp;3 \end{bmatrix} = A$

$\therefore A' = A$

Hence, A is a symmetric matrix.

(ii) We have:

$A' = \begin{bmatrix} 0 & amp;-1 & amp;1 \\ 1 & amp;0 & amp;-1 \\ -1 & amp;1 & amp;0 \end{bmatrix} = -A = \begin{bmatrix} 0 & amp;1 & amp;-1 \\ -1 & amp;0 & amp;1 \\ 1 & amp;-1 & amp;0 \end{bmatrix} = -A$

$\therefore A' = -A$

Hence, A is a skew -symmetric matrix.

40. For the matrix $A = \begin{bmatrix} 1 & amp;5 \\ 6 & amp;7 \end{bmatrix}$, verify that

(i)$(A + A')$ is a symmetric matrix

(ii) $(A - A')$ is a skew symmetric matrix

41. For the matrix $A = \begin{bmatrix} 1 & amp;5 \\ 6 & amp;7 \end{bmatrix}$, verify that

(i)$(A + A')$ is a symmetric matrix

(ii) $(A - A')$ is a skew symmetric matrix

Solutions :

Given : $A = \begin{bmatrix} 1 & amp;5 \\ 6 & amp;7 \end{bmatrix}$, then $A' = \begin{bmatrix} 1 & amp;6 \\ 5 & amp;7 \end{bmatrix}$

(i)$A + A' = \begin{bmatrix} 1 & amp;5 \\ 6 & amp;7 \end{bmatrix} + \begin{bmatrix} 1 & amp;6 \\ 5 & amp;7 \end{bmatrix}$

$= \begin{bmatrix} 2 & amp;11 \\ 11 & amp;14 \end{bmatrix}$

$\therefore (A+A')' = \begin{bmatrix} 2 & amp;11 \\ 11 & amp;14 \end{bmatrix} = A + A'$

Hence, $(A+A')$ is a symmetric matrix.

(ii)$A - A' = \begin{bmatrix} 1 & amp;5 \\ 6 & amp;7 \end{bmatrix} - \begin{bmatrix} 1 & amp;6 \\ 5 & amp;7 \end{bmatrix}$

$= \begin{bmatrix} 0 & amp;-1 \\ 1 & amp;0 \end{bmatrix}$

$(A-A')' = \begin{bmatrix} 0 & amp;-1 \\ 1 & amp;0 \end{bmatrix}$

$= -\begin{bmatrix} 0 & amp;1 \\ -1 & amp;0 \end{bmatrix} = -(A-A)'$

Hence, $(A-A')$ is a skew-symmetric matrix.

42. Find $\frac{1}{2}(A-A')$ and $\frac{1}{2}(A-A')$, when $A = \begin{bmatrix} 0 & amp;a & amp;b \\ -a & amp;0 & amp;c \\ -b & amp;-c & amp;0 \end{bmatrix}$

Solutions :

The given matrix is $A = \begin{bmatrix} 0 & amp;a & amp;b \\ -a & amp;0 & amp;c \\ -b & amp;-c & amp;0 \end{bmatrix}$, then

$A' = \begin{bmatrix} 0 & amp;-a & amp;-b \\ a & amp;0 & amp;-c \\ b & amp;c & amp;0 \end{bmatrix}$

$A + A' = \begin{bmatrix} 0 & amp;a & amp;b \\ -a & amp;0 & amp;c \\ -b & amp;-c & amp;0 \end{bmatrix} + \begin{bmatrix} 0 & amp;-a & amp;-b \\ a & amp;0 & amp;-c \\ b & amp;c & amp;0 \end{bmatrix}$

$= \begin{bmatrix} 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \end{bmatrix}$

$\therefore \frac{1}{2}(A+A') = \begin{bmatrix} 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \end{bmatrix}$

Now , $A - A' = \begin{bmatrix} 0 & amp;a & amp;b \\ -a & amp;0 & amp;c \\ -b & amp;-c & amp;0 \end{bmatrix} - \begin{bmatrix} 0 & amp;-a & amp;-b \\ a & amp;0 & amp;-c \\ b & amp;c & amp;0 \end{bmatrix}$

$= \begin{bmatrix} 0 & amp;2a & amp;2b \\ -2a & amp;0 & amp;2c \\ -2b & amp;-2c & amp;0 \end{bmatrix}$

$\therefore \frac{1}{2}(A-A') = \begin{bmatrix} 0 & amp;a & amp;b \\ -a & amp;0 & amp;c \\ -b & amp;-c & amp;0 \end{bmatrix}$

43. Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{bmatrix} 3 & amp;5 \\ 1 & amp;-1 \end{bmatrix}$

(ii) $\begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & amp;3 & amp;-1 \\ -2 & amp;-2 & amp;1 \\ -4 & amp;-5 & amp;2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & amp;5 \\ -1 & amp;2 \end{bmatrix}$

Solutions :

Let $A = \begin{bmatrix} 3 & amp;5 \\ 1 & amp;-1 \end{bmatrix}$, then $A' = \begin{bmatrix} 3 & amp;1 \\ 5 & amp;-1 \end{bmatrix}$

Now , $A + A' = \begin{bmatrix} 3 & amp;5 \\ 1 & amp;-1 \end{bmatrix} + \begin{bmatrix} 3 & amp;1 \\ 5 & amp;-1 \end{bmatrix}$

$= \begin{bmatrix} 6 & amp;6 \\ 6 & amp;-2 \end{bmatrix}$

Let $P = \frac{1}{2}(A+A') =$

$\frac{1}{2}\begin{bmatrix} 6 & amp;6 \\ 6 & amp;-2 \end{bmatrix} = \begin{bmatrix} 3 & amp;3 \\ 3 & amp;-1 \end{bmatrix}$

Now , $P' = \begin{bmatrix} 3 & amp;3 \\ 3 & amp;-1 \end{bmatrix} = p$

Thus , $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now,$A - A' = \begin{bmatrix} 3 & amp;5 \\ 1 & amp;-1 \end{bmatrix} - \begin{bmatrix} 3 & amp;1 \\ 5 & amp;-1 \end{bmatrix}$

$= \begin{bmatrix} 0 & amp;4 \\ -4 & amp;0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A') =$

$\frac{1}{2}\begin{bmatrix} 0 & amp;4 \\ -4 & amp;0 \end{bmatrix} = \begin{bmatrix} 0 & amp;2 \\ -2 & amp;0 \end{bmatrix}$

Now , $Q' = \begin{bmatrix} 0 & amp;2 \\ -2 & amp;0 \end{bmatrix} = -Q$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of$P$and Q :

$P + Q = \begin{bmatrix} 3 & amp;3 \\ 3 & amp;-1 \end{bmatrix} + \begin{bmatrix} 0 & amp;2 \\ -2 & amp;0 \end{bmatrix}$

$= \begin{bmatrix} 3 & amp;5 \\ 1 & amp;-1 \end{bmatrix} = A$

(ii) Let $A = \begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix}$, then

$A' = \begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix}$

Now $A + A' = \begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix} + \begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix}$

$= \begin{bmatrix} 12 & amp;-4 & amp;4 \\ -4 & amp;6 & amp;-2 \\ 4 & amp;-2 & amp;6 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A')$

$= \frac{1}{2}\begin{bmatrix} 12 & amp;-4 & amp;4 \\ -4 & amp;6 & amp;-2 \\ 4 & amp;-2 & amp;6 \end{bmatrix} = \begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix}$

(iii)Let $A = \begin{bmatrix} 3 & amp;3 & amp;-1 \\ -2 & amp;-2 & amp;1 \\ -4 & amp;-5 & amp;2 \end{bmatrix}$,

then $A' = \begin{bmatrix} 3 & amp;-2 & amp;-4 \\ 3 & amp;-2 & amp;-5 \\ -1 & amp;1 & amp;2 \end{bmatrix}$

Now, $A + A' = \begin{bmatrix} 3 & amp;3 & amp;-1 \\ -2 & amp;-2 & amp;1 \\ -4 & amp;-5 & amp;2 \end{bmatrix} + \begin{bmatrix} 3 & amp;-2 & amp;-4 \\ 3 & amp;-2 & amp;-5 \\ -1 & amp;1 & amp;2 \end{bmatrix}$

$= \begin{bmatrix} 6 & amp;1 & amp;-5 \\ 1 & amp;-4 & amp;-4 \\ -5 & amp;-4 & amp;4 \end{bmatrix}$

$P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & amp;1 & amp;-5 \\ 1 & amp;-4 & amp;-4 \\ -5 & amp;-4 & amp;4 \end{bmatrix}$

$= \begin{bmatrix} 3 & amp;\frac{1}{2} & amp;-\frac{5}{2} \\ \frac{1}{2} & amp;-2 & amp;-2 \\ -\frac{5}{2}7-2 & amp;2 & \end{bmatrix}$

Now , $P' = \begin{bmatrix} 3 & amp;\frac{1}{2} & amp;-\frac{5}{2} \\ \frac{1}{2} & amp;-2 & amp;-2 \\ -\frac{5}{2}7-2 & amp;2 & \end{bmatrix} = P$

Thus $P = \frac{1}{2}(A + A')$ is symmetric matrix.

Now,$A - A' = \begin{bmatrix} 3 & amp;3 & amp;-1 \\ -2 & amp;-2 & amp;1 \\ -4 & amp;-5 & amp;2 \end{bmatrix} - \begin{bmatrix} 3 & amp;-2 & amp;-4 \\ 3 & amp;-2 & amp;-5 \\ -1 & amp;1 & amp;2 \end{bmatrix}$

$= \begin{bmatrix} 0 & amp;5 & amp;3 \\ -5 & amp;0 & amp;6 \\ -3 & amp;-6 & amp;0 \end{bmatrix}$

Now , $P' = \begin{bmatrix} 3 & amp;3 \\ 3 & amp;-1 \end{bmatrix} = p$

Thus , $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now,$A - A' = \begin{bmatrix} 3 & amp;5 \\ 1 & amp;-1 \end{bmatrix} - \begin{bmatrix} 3 & amp;1 \\ 5 & amp;-1 \end{bmatrix}$

$= \begin{bmatrix} 0 & amp;4 \\ -4 & amp;0 \end{bmatrix}$

now ,$P = \begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix} = p$

Thus ,$P = \frac{1}{2}(A + A')$is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix} + \begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix}$

$= \begin{bmatrix} 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \end{bmatrix}$

Now,$Q' = \begin{bmatrix} 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \end{bmatrix} = -Q$

Thus $Q = \frac{1}{2}(A - A')$is a skew-symmetric matrix.

Representing A as the sum of P and Q

$P + Q = \begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix} + \begin{bmatrix} 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \\ 0 & amp;0 & amp;0 \end{bmatrix}$

$= \begin{bmatrix} 6 & amp;-2 & amp;2 \\ -2 & amp;3 & amp;-1 \\ 2 & amp;-1 & amp;3 \end{bmatrix} = A$